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Superlattice Crystal Accelerator: Acceleration Beyond GeV/m

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Abstract

Here, an idea of using a visible light wave to accelerate relativistic particles via the inverse FEL mechanism is explored. A strain modulated crystal structure – the superlattice, plays the role of a microscopic undulator providing very strong ponderomotive coupling between the beam and the light wave. Purely classical treatment of relativistic protons channeling through a superlattice is performed in a self consistent fashion involving the Maxwell wave equation for the accelerating electromagnetic field and the relativistic Boltzmann equation for the protons. It yields the accelerating efficiency in terms of the negative gain coefficient for the amplitude of the electromagnetic wave – the rate the energy is extracted from the light by the beam. Presented analytic formalism allows one to find the acceleration rate in a simple closed form, which is further evaluated for a model beam – optical cavity system to verify feasibility of this scheme.

I. INTRODUCTION

The main idea of using a modulated crystal structure as an undulator is illustrated schematically in Figure 1. A beam of relativistic particles while channeling through the crystal follows well defined trajectories. The particles are periodically accelerated perpendicular to their flight path as they traverse the channel. The undulator wavelengths typically fall in the range 50–500 Å, far shorter than those of any macroscopic undulator.

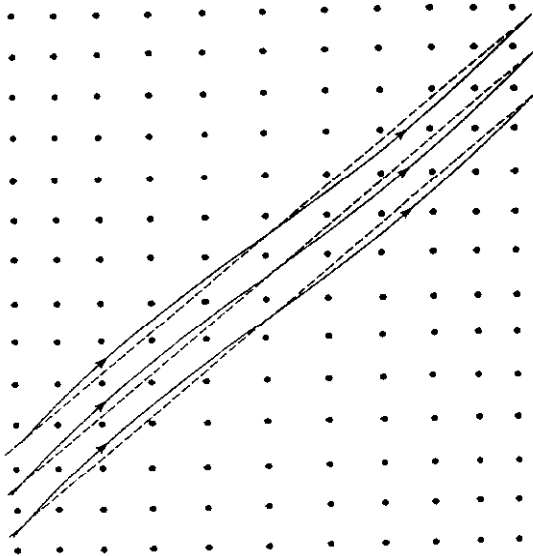


Figure 1 Center of the channeling trajectory in [110] direction in a strain-modulated superlattice.

Furthermore, the electrostatic crystal fields involve the line averaged nuclear field and can be two or more orders of magnitude larger than the equivalent fields of macroscopic magnetic undulators. Both of these factors hold the promise of greatly enhanced coupling between the beam and the accelerating electromagnetic wave.

II. SUPERLATTICE CHANNELING

One can describe a high intensity proton beam in terms of a classical distribution function, $f(p, x, t)$, governed by the relativistic Boltzmann equation. The transverse dynamics of relativistic protons propagating in a strain modulated superlattice is modeled by a harmonic crystal field potential¹ and leads to generation of a transverse current. This couples the Vlasov equation to the Maxwell wave equation. Therefore, presented problem reduces to a self consistent solution of the Vlasov and the wave equations.

Collective behavior of a particle beam channeling along the z axis can be described in terms of the relativistic Vlasov equation

$$\frac{\partial f}{\partial t} + \frac{1}{m\gamma} \left(p^\alpha - \frac{e}{c} A^\alpha \right) \frac{\partial f}{\partial x^\alpha} + \left[\frac{e}{mc\gamma} \left(p^\beta - \frac{e}{c} A^\beta \right) \frac{\partial A^\beta}{\partial x^\alpha} - e \frac{\partial \phi}{\partial x^\alpha} \right] \frac{\partial f}{\partial p^\alpha} = 0. \quad (1)$$

Here A is a vector potential of an electromagnetic field and ϕ is a phenomenological harmonic crystal-field potential², which describes both transverse focusing of the beam and longitudinal modulation of the minimum of the harmonic potential well.

$$\phi = \phi_0 + \frac{1}{2} \phi_1 (x - x_1 \cos gz)^2, \quad (2)$$

where $g = 2\pi/l$, l is the strain modulation periodicity and x_1, ϕ_1, ϕ_0 are parameters of the potential.

Eq. (1) will be treated iteratively and only linear terms in the A -field will be retained. In the 0-th order solution $A = 0$, and the corresponding distribution function $f = f^{(0)}$ is obtained from the solution of

$$\frac{\partial f^{(0)}}{\partial t} + \frac{p^\alpha}{m\gamma} \frac{\partial f^{(0)}}{\partial x^\alpha} - e \frac{\partial \phi}{\partial x^\alpha} \frac{\partial f^{(0)}}{\partial p^\alpha} = 0. \quad (3)$$

A class of solutions, $f^{(0)}$, describing a beam with a sharply peaked initial momentum distribution, Λ can be constructed as follows

$$f^{(0)} = n_0 \delta(x - x^{(0)}) \delta(p_x - p_x^{(0)}) \Lambda(p_z - p_0), \quad (4)$$

where n_0 is a concentration of particles per unit area of the channeling plane and the steady state trajectory is described as follows

$$x^{(0)} = \frac{x_1}{1 - U^2} \cos(gz) \quad (5)$$

$$p_x^{(0)} = p_z \frac{\partial x^{(0)}}{\partial z}$$

Here $U = g/k_\beta$, where, k_β is a focusing strength of the crystal channel given explicitly below

$$k_\beta^2 = \frac{e\phi_1}{p_z v_z} \quad (6)$$

We have assumed that only the transverse component of the A-field is present and $A_x \equiv A(z, t)$. We seek a perturbed solution, $f^{(1)}$, in the following form

$$f^{(1)} = n_0 \delta(x - x^{(0)}) \delta(p_x - p_x^{(0)}) h(z, p_z, t), \quad (7)$$

where h describes bunching of particles due to the presence of the A-field. Substituting Eqs.(4)–(7) and (2) into Eq.(1) leads to the following kinetic equation for h

$$\frac{\partial h}{\partial t} + \frac{p_z}{m\gamma} \frac{\partial h}{\partial z} + \frac{e}{c} \frac{p_z}{m\gamma} \frac{g x_1}{1 - U^2} \frac{\partial A}{\partial z} \frac{\partial \Lambda}{\partial p_z} \sin gz = 0. \quad (8)$$

The inhomogeneous term in the above equation plays the role of a driving force representing acceleration of the particles by the ponderomotive force due to the transverse motion (induced by the crystal field) in the presence of the A-field. The resulting transverse current couples Eq.(8) to the following wave equation

$$\left(\frac{\partial^2}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) A = \frac{4\pi n e}{c} \int_{-\infty}^{\infty} dp_z h \frac{p_z}{m\gamma} \frac{g x_1}{1 - U^2} \sin gz, \quad (9)$$

resulting in a closed system of equations for h and A . Here the A-field can be identified as a sum of the macroscopic driving field and a self consistent electromagnetic field propagating in the crystal structure. We start with a single plane wave solution of arbitrary ω and k propagating in free space along the z axis in both directions and use it as a 0-th order iteration step

$$A_{\pm}^{(0)} = A_0 e^{-i\omega t \pm i k z} \quad (10)$$

Putting $A = A_{\pm}^{(0)}$ (left and right propagating waves) in Eq.(9), one can solve it analytically for $h = h_{\pm}^{(1)}$ by constructing a Green's function with the appropriate boundary conditions built in it. The solutions for $A_{\pm}^{(1)}(z)$ can be written explicitly in terms of the Green's function for the Helmholtz equation. On the other hand,

one can model the effect of coupling by adding a small complex part $ik^{\pm} = \alpha^{\pm} + i\beta^{\pm}$ to the k -vector; here α is a gain/loss coefficient and β describes a small shift in the phase velocity of the optical mode. Deamplification of the back traveling wave can be summarized by the following expression

$$\alpha^- = -\frac{\pi}{4} n \int_{-\infty}^{\infty} dp_z Q^2 \frac{p_z}{m\gamma} \frac{\partial \Lambda}{\partial p_z} L\Gamma(v^-), \quad (11)$$

where

$$v^- = m\gamma kc/p_z + k - g, \quad (12)$$

and

$$Q = \frac{e}{c} \frac{g x_1}{1 - U^2} \quad (13)$$

Here,

$$\Gamma(x) \equiv \left(\frac{\sin(xL/2)}{xL/2} \right)^2 \quad (14)$$

is the characteristic form occurring in diffraction theory, with the principal maximum at $x = 0$.

III. ACCELERATION RATE

Imposing resonant condition, $v^- = 0$, in Eq.(11) fixes the wave vector of the optical mode as follows

$$g = m\gamma kc/p_z + k \quad (15)$$

One can notice that, apart from a slowly varying function Q , the remaining functions occurring in the integrand in Eq.(11), namely, Λ and Γ are sharply peaked functions of momentum characterized by the respective widths:

$$\left(\frac{\Delta p}{p} \right)_A \quad \text{and} \quad \left(\frac{\Delta p}{p} \right)_\Gamma = \frac{l}{L} \quad (16)$$

Now one can compare relative sharpness of both functions; Λ and Γ . Typical value of the relative momentum spread is of the order of 10^{-4} . Assuming superlattice modulation of 500Å and crystal length of 5 cm allows one to evaluate the width of Γ . Both characteristic widths can be summarized as follows

$$\left(\frac{\Delta p}{p} \right)_A = 10^{-4}, \quad \left(\frac{\Delta p}{p} \right)_\Gamma = 10^{-6}. \quad (17)$$

The integration in Eq.(11) is carried out assuming that the sharper function, namely Γ , is approximated by the δ -function. This reduces the gain/loss coefficient to the following simple expression

$$\alpha = -\frac{\pi}{2} n \frac{Q^2}{m} \gamma l \left(\frac{\Delta p}{p} \right)_A^{-2} \quad (18)$$

The above final result will serve as a starting point for further feasibility discussion.

IV. THREE WAVE MIXING

Spontaneous bunching of the proton beam channeling through a superlattice and interacting with the electromagnetic wave results in energy flow from the wave to the beam. This particular kind of particle density fluctuation, h , has the form of a propagating plane wave of the same frequency, ω , as the emitted electromagnetic wave. The phase velocity of the moving bunch matches the velocity of protons in the beam. Therefore, the quantity $\gamma m \omega / p_z \equiv k_b$ represents the wave vector of the propagating particle density bunch. Keeping in mind that the periodicity of the undulator represents a static wave with a wave vector g , and that k is the wave vector of the electromagnetic wave, we can analyze our results in the language of three wave mixing³.

Furthermore "momentum" conservation of all three modes yields the $v^- = 0$ conditions. The last condition, $k_b = g - k$, is equivalent to a momentum "recoil" between the particle density "bunch" and the electromagnetic wave (deamplification of the backward propagating wave), where a four momentum $(0, g)$ is transferred from the backward propagating wave to the forward moving proton bunch.

V. FEASIBILITY ASSESSMENT

We will discuss the feasibility of the proposed scheme by considering (110) planar channeling in a strain modulated Si crystal⁴. We write the undulator period as $\ell = Nd$, where $d = 1.92 \text{ \AA}$ is the spacing between successive lattice planes and N is the number of such planes. The strain modulation, of course, requires a second component, such as Ge; however, we will use the parameters of Si for convenience.

Relativistic particles while channeling along the path undergo transverse harmonic oscillations from the crystal field potential, an analog of the betatron oscillations, with the characteristic frequency $\omega_b = \sqrt{e\phi_1}/m$. One can see from Eq.(13) that if the angular velocity of a particle traversing the strain modulated path, $\omega = 2\pi v_{\parallel}/\ell$, approaches $\omega_b/\sqrt{\gamma}$ (Doppler shifted betatron frequency), the undulator parameter, Q , has a resonance ($U \rightarrow 1$), which would enormously enhance the gain/loss coefficient. However, the excessive growth of the undulator parameter would soon result in a rapid dechanneling of the particles. One can see this easily if Q is rewritten in the following form

$$Q = \frac{e}{c} \frac{v_{\perp}^{\max}}{v_{\parallel}}, \quad (19)$$

where v_{\perp}, v_{\parallel} are transverse and longitudinal components of the particle velocity, respectively.

For small values of γ ($U \approx 1$), the following simple physical criterion allows one to estimate the maximum value of Q . Dechanneling will occur if the transverse kinetic energy of the particle exceeds the binding energy of the harmonic potential (a particle leaves the channel). If the maximum transverse velocity of a channeling particle is v_{\perp} and a is the distance between adjacent channels (for (110) channeling in Si $a = 5 \text{ \AA}$), the above condition can be written as follows:

$$v_{\perp}^2 \geq \frac{e\phi_1}{m\gamma} \left(\frac{a}{2}\right)^2. \quad (20)$$

The equality sign in Eq.(20) along with Eq.(19) fix the maximum allowed value of the undulator parameter as

$$Q^{\max} = \frac{e}{c} \frac{a}{2} \sqrt{\frac{e\phi_1}{m\gamma c^2} \frac{\gamma}{\gamma^2 - 1}}. \quad (21)$$

The above expression can be evaluated for relativistic protons channeling through our model superlattice as

$$Q^{\max} = 7.5 \times \sqrt{\frac{\gamma}{\gamma^2 - 1}} \times 10^{-24} \text{ cm}^{1/2} \text{ g}^{1/2}. \quad (22)$$

Now, one can evaluate Eq.(11) assuming only one proton – by assigning n to be an inverse area of the channeling plane per one particle for typical values of the beam concentration⁵, $n = 10^{16} \text{ cm}^{-2}$. This way α describes the rate of optical amplitude depletion per one particle – the acceleration rate. Assuming γ of 2, $\ell = 500 \text{ \AA}$ and

$$\left(\frac{\Delta p}{p}\right)_A = 10^{-4} \quad (23)$$

yields the following value of the acceleration rate

$$\alpha = 3.53 \times 10^{-4} \text{ cm}^{-1}. \quad (24)$$

The nominal acceleration efficiency in units of eV/cm will, obviously, depend on the energy density of the actual optical cavity. A state of the art visible laser⁶ would provide E-field of 10^{11} V/m . As concluded elsewhere⁷ it yields the accelerating gradient of 3.5 GeV/m.

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